Solid-State and Soft Matter Photonics

Spring, 2020 **Problem Set No.1**

Professor Y.Tomita **Due: June 03, 2020**

1. (20 points) **Symmetry of the conductivity tensor**
   If the only currents in a medium are due to conduction and if the medium is linear, the following simple relation between the current density $\boldsymbol{J}$ and an applied electric field $\boldsymbol{E}$ written by a 3×1 column vectors is hold:

   $$\boldsymbol{J} = \tilde{\sigma} : \boldsymbol{E},$$

   where $\tilde{\sigma}$ is the second-rank conductivity tensor written by a 3×3 matrix and ":" denotes the tensor product.

   (a) (10 points) Prove mathematically that any second-rank tensor A (expressed by a 3×3 matrix) can be always expressed by the sum of a symmetric part A^s and an antisymmetric one A^a, where $(A^s)_{ij} = (A^s)_{ji}$ $(i \neq j)$ and $(A^a)_{ij} = -(A^a)_{ji}$ $(i \neq j)$ with $(A^a)_{ii} = 0$.

   (b) (10 points) The power dissipated P in a conducting medium per volume is given by

   $$P = \boldsymbol{E} \cdot \boldsymbol{J} = \boldsymbol{E} \cdot \tilde{\sigma} : \boldsymbol{E}.$$

   Show mathematically that only the symmetric part of $\tilde{\sigma}$ contributes to the power dissipation.

2. (10 points) **Time-reversal symmetry of the electric polarization vector**
   Show that the electric polarization vector $\boldsymbol{P}(\boldsymbol{r}, -\omega)$ corresponds to the time-reversal form of $\boldsymbol{P}(\boldsymbol{r}, t)$.

3. (10 points) **Symmetry of the complex electric susceptibility tensor**
   Show that the second-rank complex electric susceptibility tensor $\tilde{\chi}_e(\boldsymbol{r}, \omega)$ written by a 3×3 matrix is Hermitian, *i.e.*, $[\tilde{\chi}_e(\boldsymbol{r}, \omega)]^*_{ij} = [\tilde{\chi}_e(\boldsymbol{r}, \omega)]_{ji}$ $(i \neq j)$, when $\Re[i\omega \boldsymbol{E}(\boldsymbol{r}, \omega) \cdot \epsilon_0 \tilde{\chi}^*_e(\boldsymbol{r}, \omega) \boldsymbol{E}^*(\boldsymbol{r}, \omega)] = 0$ ($\Re$ means a real part) in the case of no free charge current (*i.e.*, lossless media).

*These problems can be solved in reference to the lecture notes #1 and 2.