Solid-State and Soft Matter Photonics
Spring 2020 **Problem Set No.2**
Professor Y.Tomita **Due: July 01, 2020**

1. (20 points) **Dirac identity**
   Prove the following Dirac identity:

$$\left.\frac{1}{r \mp i\epsilon}\right|_{\epsilon\to 0} = \mathcal{P}\frac{1}{r} \pm i\pi\delta(r),$$

   where $\mathcal{P}$ is the Cauchy's principal value. (Hint: Consider a complex function $f(z)$ that is analytical in the upper half-plane and approaches zero as $|z| \to \infty$.)

2. (10 points) **Kramers-Kronig transformations**
   Show that Kramers-Kronig transformations derived in the previous lecture can take other forms as given by

$$\begin{aligned}\chi'(\omega) &= \frac{2}{\pi}\mathcal{P}\int_0^\infty d\nu \frac{\nu\chi''(\nu)}{\nu^2-\omega^2}, \\ \chi''(\omega) &= -\frac{2\omega}{\pi}\mathcal{P}\int_0^\infty d\nu \frac{\chi'(\nu)}{\nu^2-\omega^2},\end{aligned}$$

   where $\mathcal{P}$ denotes the Cauchy's principal value.

3. (20 points) **Quantum mechanical harmonic oscillators**
   (a) (10 points) In the lecture we studied the oscillator strength f_{nl} for an atom in a classical light field. The following sum rule for f_{nl} is found:

$$\sum_{n\neq l} f_{nl} = 1.$$

   Derive the oscillator strength for the transition between the n and l states of a quantum mechanical harmonic oscillator.
   Hint: You can use main results of quantum mechanical harmonic oscillators, which can be found in books on introductory quantum mechanics, for example, J. J. Sakurai, *Modern Quantum Mechanics*, p.92, (Benjamin/Cummings, 1985), R. Shankar, *Principles of Quantum Mechanics*, p.207, (Plenum Press, 1980).

   (b) (10 points) Prove that the above sum rule is also applicable to quantum mechanical harmonic oscillators.